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M E X I C O

Would Rational Voters Acquire Costly Information?

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Abstract

We analyze an election in which voters are uncertain about which of two alternatives is better for them. Voters can, however, acquire some costly information about the alternatives. As the number of voters increases, individual investment in political information declines to zero. However, the election outcome is likely to correspond to the interest of the majority if the marginal cost of information acquisition approaches zero as the information acquired becomes nearly irrelevant. Under certain conditions, the election outcome corresponds to the interests of the majority with probability approaching one. Thus, "rationally ignorant" voters are consistent with a well-informed electorate. *JEL* D72, D82.

Keywords: voting, information acquisition, information aggregation.

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1 Introduction

One of the most influential contributions of Anthony Downs's *An Economic Theory of Democracy* to the economic modeling of politics is the concept of "rational ignorance." Given that each individual voter has a negligible probability of affecting the outcome in a large election, voters will not have an incentive to acquire political information before voting. In a situation in which discovering their interests or "true views" takes time and effort from individual citizens, the result may be a failure of democracy to produce a result consistent with the interests of the majority. In Downs's words,

> If all others express their true views, he [the voter] gets the benefit of a well-informed electorate no matter how well-informed he is; if they are badly informed, he cannot produce those benefits himself. Therefore, as in all cases of individual benefits, the individual is motivated to shirk his share of the costs: he refuses to get enough information to discover his true views. Since all men do this, the election does not reflect the true consent of the governed. (Downs 1957, p. 246)

We can actually draw a distinction between two versions of the rational ignorance hypothesis. The "weak version" is that individual voters, realizing that each vote has a negligible probability of affecting the outcome of the election, invest very little or no effort in acquiring political information. The "strong version" is that the election outcome itself will not be more likely to reflect the interests of the majority than, say, a fair coin toss. In this paper, we develop a formal model that is consistent with the weak version of the rational ignorance hypothesis, but contradicts the strong version.

A good deal of the literature on the influence activities of interest groups assumes explicitly or implicitly that a decisive fraction of the electorate is uninformed because individual voters have little incentive to get political information (see e.g. Becker 1983, Baron 1994, and Grossman and Helpman 1996). Becker (1985) argues that efficiency may be restored in the voting market because of the activity of influence groups. Coate and Morris (1995) point out that the reelection motive may induce incumbent politicians to behave efficiently unless voters are uncertain about politicians' types. (In their view, and Becker's, efficiency does not mean that transfers from the majority to interest groups do not occur; it only means that those transfers are carried out with minimum dead weight costs.) Closer to our point, Wittman (1989) calls into question the idea that the costs of information fall on the voter instead of on political entrepreneurs.

We provide a different rationale for elections to reflect the interests of the majority. In our model, there are no interest groups or active politicians. Voters do not have access to free information. Instead, they may acquire some information, at a cost. Crucially, acquiring poor information is cheap. We show that, as the number of voters increases, voters acquire less and less information. However, under some conditions detailed below, the outcome of the election is very likely to correspond to the interests of a majority of voters. Thus, the electorate may be quite well-informed even if individual voters are (at least asymptotically) rationally ignorant.

We study an election in which "moderate" or "swing" voters do not know which of two alternatives is better for them. Voters may acquire a costly signal about the alternatives. The signal is correct with probability $1/2 + x$, where x is chosen by the voter. We refer to x as the quality of the signal. The cost of acquiring the signal is given by some convex function $C(x)$. Our first three results describe information acquisition and information aggregation in the context of this model.

Theorem 1 shows that the quality of information acquired by individual voters goes to zero as the size of the electorate increases. However, if $C'(0) = 0$, then the quality of information is positive for an arbitrarily large electorate. The reason is simple: the probability of being pivotal is not exactly zero. (If the probability of being pivotal were zero, instrumentally rational voting behavior would be unconstrained.) An example consistent with our assumptions is $C(x) = x^\gamma$, with $\gamma > 1$.

Theorem 2 provides an estimate of the limit probability of choosing the best alternative. If $C'(0) = 0$ and $C''(x)$ is bounded, this probability is strictly larger than 1/2. If $C'(0) = C''(0) = 0$, this probability is actually one. In the example above, this is the case as long as $\gamma > 2$. Successful information aggregation is possible because the information acquired by each moderate voter goes to zero but it does so slowly enough to allow the effect of large numbers to kick in.

The conditions for successful information aggregation may seem restrictive. But it is reasonable to believe that voters are involuntarily exposed to a flow of political information in the course of everyday activities – a point already acknowledged by Downs (1957, p. 245), who relies on the unwillingness of voters to assimilate even freely available information in order to support the rational ignorance hypothesis. If the function C simply reflects the cost of "paying a little attention," the conditions for successful information aggregation do not appear unduly restrictive.

Theorem 3 shows that elections with information acquisition will be almost always very close. On one hand, elections must be close to keep individual voters acquiring some information. On the other hand, the fact that voters acquire vanishingly little information keeps elections close even as the number of voters increases.

Our last result, Theorem 4, considers a variant of our model in which voters receive an informative signal even if they do not invest any effort into acquiring political information. It comes as a mild surprise that, as long as $C'(0) = 0$, voters will devote some positive effort to information acquisition. (Of course, whether or not there is information acquisition becomes asymptotically irrelevant, as the probability that a moderate voter casts a vote in agreement with his "true views" is bounded below by a number larger than one half.)

Taken together, we consider our results as an argument in favor of the idea that elections tend to serve well the interests of the majority, even if the rational ignorance hypothesis fits well as a description of individual voters.

Our model is related to the literature on information aggregation in elections inspired by Condorcet's jury theorem (e.g. Miller 1986, Young 1988, Austen-Smith and Banks 1996, Feddersen and Pesendorfer 1997, Duggan and Martinelli 2001). This literature typically assumes that there is some information dispersed among the voters, while in our paper the distribution of information arises endogenously through the actions of voters. Our result on close elections is similar to a result by Feddersen and Pesendorfer, though it is obtained for quite different reasons (see the discussion in Section 5). It is interesting to note that voting behavior with private information leads to close elections under a variety of assumptions.

Recently, Persico (1999) has proposed another model of endogenous information in collective decision making. In Persico's model, the quality of the signal is given; voters can either acquire or not acquire information. As a consequence, in his model it is not possible to have arbitrarily large numbers of voters acquiring arbitrarily poor information. Persico is concerned with the optimal design of committees, i.e. the optimal selection of committee size and voting rule, while we consider an environment where majority rule is optimal and concern ourselves with the positive issue of endogenous production and aggregation of information in large elections.

This paper also bears relation with and is partly inspired by the work of scholars of public opinion who have noted that, in the aggregate, voters seem surprisingly well-informed about the choices they make (e.g. Converse 1990 and Stimson 1990). The apparent paradox of nearly uninformed voters and a well-informed electorate is aptly described by the following passage:

> There is no paradox in here: the combination of some political knowledge with scores approaching zero on information tests simply attests to the astronomical size of the potential universe of political information. (Converse 1990, p. 372)

2 The Model

We analyze an election with two alternatives, A and B. There are $2n+1$ voters ($i = 1, \ldots, 2n+1$). A voter's utility depends on the chosen alternative $d \in \{A, B\}$, a preference parameter $t \in \{t_A, t_M, t_B\}$, the state $z \in \{z_A, z_B\}$, and the quality of information acquired by the voter before the election $x \in [0, 1/2]$. Acquiring information of quality x has a utility cost given by $C(x)$, so the utility of a voter can be written as

$$U(d, t, z) - C(x).$$

At the beginning of time, nature selects the state and the type of each voter. Both states are equally likely ex ante. Each voter's type is equal to t_A with probability ϵ, to t_B with probability ϵ, and to t_M with probability $1 - 2\epsilon$, where $0 < \epsilon < 1/2$. Voters' types are independent from each other and from the realization of the state.

Each voter knows her preference type but is uncertain about the type of other voters. Voters are also uncertain about the realization of the state. After learning her type, a voter decides the quality of her information. After deciding on x, the voter receives a signal $s \in \{s_A, s_B\}$. The probability of receiving signal s_A in state A is equal to the probability of receiving signal s_B in state B and is given by $1/2 + x$. That is, the likelihood of receiving the "right" signal is increasing in the quality of information acquired by the voter; if the voter acquires no information the signal is uninformative. Signals are private information.

The election takes place after voters receive their signals. A voter can either vote for A or vote for B. The alternative with most votes is chosen.

Let

$$v(t, z) = U(A, t, z) - U(B, t, z).$$

We assume that

$$\begin{array}{rcccl} v(t_M, z_A) & = & -v(t_M, z_B) & = & r, \\ v(t_A, z_A) & = & v(t_A, z_B) & = & q, \\ v(t_B, z_A) & = & v(t_B, z_B) & = & -q \end{array}$$

where r and q are two positive real numbers. Voters of type t_A and t_B are "extremists," who favor alternative A or alternative B regardless of the possible circumstances or states. Voters of type t_M are "moderates," willing to support alternative A or alternative B depending on the circumstances.

The cost function C is strictly increasing, strictly convex, and twice continuously differentiable on $(0, 1/2)$. We assume that $C(0) = 0$, so that acquiring no information is costless. Note that $C'(0) \in [0, \infty)$. If $C''(x)$ grows unboundedly as x goes to zero, we use the notation $C''(0) = \infty$. Thus, $C''(0) \in [0, \infty]$.

After describing the environment, we turn now to the description of strategies and the definition of equilibrium in the model. A pure strategy is a pair a_x, a_v, where

$$a_x : \{t_A, t_M, t_B\} \to [0, 1/2]$$

is a mapping from a voter's type to a quality of information x, and

$$a_v : \{t_A, t_M, t_B\} \times \{s_A, s_B\} \to \{A, B\}$$

is a mapping from a voter's type and the signal received to a decision to vote for A or for B. A mixed strategy for voter i is a probability distribution α_i over the set of pure strategies.

A *voting equilibrium* $\overline{\alpha}$ ($\alpha_i = \alpha$ for all i) is a symmetric Nash equilibrium in which no voter uses a weakly dominated strategy.

Clearly, an equilibrium strategy will only assign positive probability to pure strategies such that $a_x(t_A) = a_x(t_B) = 0$ and, from elimination of weakly dominated strategies, $a_v(t_A, s) = A$, $a_v(t_B, s) = B$ for $s \in \{s_A, s_B\}$, so we can restrict our attention to pure strategies satisfying those constraints. It remains to determine the equilibrium behavior of moderate voters.

Let $P_{\alpha_1,\ldots,\alpha_{2n+1}}(A|z_A)$ and $P_{\alpha_1,\ldots,\alpha_{2n+1}}(B|z_B)$ be the probability of alternative A winning the election if the state is z_A and the probability of alternative B winning the election if the state is z_B, for a given strategy profile. Let $E_{\alpha_i}(C(x_i))$ be the expected cost of information acquisition for voter i given her own strategy, conditional on her type being t_M. Then, the ex ante utility for voter i is given by

$$(1-2\epsilon)\left[\frac{1}{2}P_{\alpha_1,\ldots,\alpha_{2n+1}}(A|z_A) + \frac{1}{2}P_{\alpha_1,\ldots,\alpha_{2n+1}}(B|z_B)\right] r + (1-2\epsilon)E_{\alpha_i}(C(x_i))$$

plus some constant term which we ignore hereafter. We refer to the term in brackets as the *probability of choosing the right alternative*. We are particularly interested in the limit value of this probability as the size of the electorate increases.

3 Rational Ignorance

In this section we describe the equilibrium behavior of moderate voters. We show that, according to the weak version of the rational ignorance hypothesis, in large elections voters acquire vanishingly little information or no information at all.

Define

$$G(x) = \frac{(2n)!}{n!n!}\left(\frac{1}{4} - (1-2\epsilon)^2 x^2\right)^n r - C'(x).$$

Intuitively, this expression gives us the marginal benefit of acquiring quality of information x for a given voter when every other voter is acquiring x. The first term in the definition of G is the probability that a given voter is pivotal multiplied by the gain in reaching the right decision. The second term is the marginal cost of quality of information x. Note that G is strictly decreasing.

Let

$$x_M = \begin{cases} 0 & \text{if } G(0) \leq 0 \\ 1/2 & \text{if } G(1/2) \geq 0 \\ G^{-1}(0) & \text{otherwise.} \end{cases}$$

The first term in the definition of G is strictly positive and converges to zero as n goes to infinity for any sequence of $x \in [0, 1/2]$. Thus, if $C'(0) = 0$, we get $G(x) > 0$ for every x and then $x_M > 0$. However, if we let n go to infinity while keeping ϵ, r and the function C constant, x_M should converge to 0.

If $C'(0) > 0$, let $n(r, C)$ be the minimum n such that

$$\frac{(2n)!}{n!n!}\left(\frac{1}{4}\right)^n r \leq C'(0).$$

Note that for any $n \geq n(r, C)$, we get $x_M = 0$.

We have

Theorem 1
(i) If $C'(0) = 0$, there is a unique voting equilibrium. In this equilibrium, the pure strategy given by $a_x(t_M) = x_M$, $a_v(t_M, s_A) = A$ and $a_v(t_M, s_B) = B$ is played with probability one.
(ii) If $C'(0) > 0$ and $n \geq n(r, C)$, every equilibrium assigns probability one to the set of pure strategies such that $a_x(t_M) = 0$.

Proof. Suppose that every voter other than i adopts the strategy α, and let $P_\alpha(\text{piv}|z_A)$ and $P_\alpha(\text{piv}|z_B)$ be the probabilities that n voters other than i vote for A and n voters other than i vote for B in state z_A and in state z_B, respectively. The expected utility for voter i of adopting the pure strategy $a_x(t_M) = x$, $a_v(t_M, s_A) = A$ and $a_v(t_M, s_B) = B$ for any $x \in [0, 1/2]$ is given by $(1 - 2\epsilon)$ times

$$(1) \qquad \left[\frac{1}{2}P_\alpha(\text{piv}|z_A)\left(\frac{1}{2} + x\right) + \frac{1}{2}P_\alpha(\text{piv}|z_B)\left(\frac{1}{2} + x\right)\right] r - C(x)$$

plus a term that does not depend on the action chosen by i. Note that the expected utility is a strictly concave function of x.

We can show that, in equilibrium, it has to be the case that the pure strategy with $a_v(t_M, s_A) = A$ and $a_v(t_M, s_B) = B$ is strictly superior to any other pure strategy for a given choice $x > 0$ of information quality. For suppose that it is not superior to the pure strategy with $a_v(t_M, s_A) = A$ and $a_v(t_M, s_B) = A$ (other cases are treated similarly). Then

$$P_\alpha(\text{piv}|z_A)\left(\frac{1}{2} + x\right) + P_\alpha(\text{piv}|z_B)\left(\frac{1}{2} + x\right) \leq P_\alpha(\text{piv}|z_A),$$

that is

$$\frac{P_\alpha(\text{piv}|z_A)}{P_\alpha(\text{piv}|z_B)} \geq \frac{1/2 + x}{1/2 - x} > 1.$$

Then, for every choice of information quality, every pure strategy such that $a_v(t_M, s_A) = B$ is strictly dominated by the pure strategy with $a_v(t_M, s_A) = A$ and $a_v(t_M, s_B) = A$. That is, voter i assigns probability one to the set of pure strategies with $a_v(t_M, s_A) = A$. Now let $\beta(x)$ be the probability that i plays a pure strategy with $a_v(t_M, s_B) = B$ and with quality of information smaller or equal than x, as induced by voter i's strategy. Let p_A and p_B be the probabilities with which voter i votes for A in state z_A and for B in state z_B, as induced by voter i strategy. Then $p_A = (1-2\epsilon)(1-\int_0^{1/2}(1/2-x)d\beta(x))+\epsilon$ and $p_B = (1-2\epsilon)(\int_0^{1/2}(1/2+x)d\beta(x)) + \epsilon$. It follows that $|p_A - 1/2| \geq |p_B - 1/2|$. But then, $p_A^n(1-p_A)^n \leq p_B^n(1-p_B)^n$. Since equilibrium is symmetric, we get $P_\alpha(\text{piv}|z_A) \leq P_\alpha(\text{piv}|z_B)$, a contradiction.

From the previous paragraph, we can restrict our attention to pure strategies with $a_x(t_M) = x$, $a_v(t_M, s_A) = A$ and $a_v(t_M, s_B) = B$ for any $x > 0$. From equation (1), if any such pure strategy is optimal for voter i, it is the unique optimal pure strategy among strategies with $a_x(t_M) = x$, $a_v(t_M, s_A) = A$ and $a_v(t_M, s_B) = B$ for any $x \geq 0$. Moreover, the argument in the previous paragraph shows that $P_\alpha(\text{piv}|z_A) = P_\alpha(\text{piv}|z_B)$. But this implies that all pure strategies with no information acquisition have the same expected payoff. Thus, if there is some information acquisition, it has to be the case that the voting equilibrium is a pure strategy equilibrium with $a_x(t_M) = x^*$, $a_v(t_M, s_A) = A$ and $a_v(t_M, s_B) = B$ for some $x^* > 0$.

Now, suppose that every voter other than i adopts the pure strategy with $a_x(t_M) = \tilde{x}$, $a_v(t_M, s_A) = A$ and $a_v(t_M, s_B) = B$ for some $\tilde{x} \geq 0$. Then the probabilities of voter i being pivotal in states z_A and z_B are

$$\begin{aligned} P(\text{piv}|z_A) &= \binom{2n}{n}(\epsilon + (1-2\epsilon)(1/2+\tilde{x}))^n(\epsilon + (1-2\epsilon)(1-1/2-\tilde{x}))^n \\ &= \frac{(2n)!}{n!n!}\left(\frac{1}{4} - (1-2\epsilon)^2\tilde{x}^2\right)^n \\ &= P(\text{piv}|z_B). \end{aligned}$$

Replacing these probabilities in equation (1), we get that the expected utility for voter i of adopting the pure strategy with $a_x(t_M) = x$, $a_v(t_M, s_A) = A$ and $a_v(t_M, s_B) = B$ is a positive affine function of

$$\frac{(2n)!}{n!n!}\left(\frac{1}{4} - (1-2\epsilon)^2\tilde{x}^2\right)^n \left(\frac{1}{2} + x\right) r - C(x).$$

The first derivative of this expression with respect to x is

$$H(x, \tilde{x}) = \frac{(2n)!}{n!n!}\left(\frac{1}{4} - (1-2\epsilon)^2\tilde{x}^2\right)^n r - C'(x).$$

The second derivative is negative for $x > 0$. Note that $H(x, x) = G(x)$. Thus, the distribution that gives probability one to the pure strategy $a_x(t_M) = x_M$, $a_v(t_M, s_A) = A$ and $a_v(t_M, s_B) = B$ constitutes a voting equilibrium. Moreover, this is the only equilibrium in which information acquisition is possible.

To check that, if $C'(0) = 0$, there is no equilibrium in which there is no information acquisition, note that the probability of being pivotal is positive for any choice of strategy by other voters. Thus, from equation (1), the marginal benefit of acquiring information is larger than the marginal cost for x sufficiently close to 0. Finally, if $C'(0) > 0$, we know that $x_M = 0$ for $n \geq n(r, C)$. It follows that there is no equilibrium with information acquisition for $n \geq n(r, C)$.

□

4 Information Aggregation

In this section, we let n go to infinity while keeping the other parameters of the model (ϵ, q, r) and the function C constant. We study the limit behavior of the probability of choosing the right alternative along the sequence of voting equilibria thus obtained.

From the previous section, we know that if $C'(0) > 0$, there is no information acquisition for n large enough. Thus, the probability of choosing the right alternative converges to $1/2$ – the only possibility for an uninformed electorate since the two states are equally likely. However, if $C'(0) = 0$, moderate voters acquire some information for every n. In this section we show that, if $C'(0) = C''(0) = 0$, the quality of information acquired by moderate voters declines slowly enough to allow the probability of choosing the right alternative to converge to one. In other words, even though in the limit voters are rationally ignorant, the electorate is quite well informed. If $C'(0) = 0$ and $C''(0) = c \in (0, \infty)$, some information aggregation is still possible; in this case the limit value of the probability of choosing the right alternative increases with r/c (the gain of choosing the right alternative divided by the limit of the second derivative of the cost function). Finally, if $C'(0) = 0$ but $C''(0) = \infty$, the quality of information acquired by voters declines very fast so the probability of choosing the right alternative converges to $1/2$.

As an example, consider the cost function $C(x) = x^\gamma$, with $\gamma > 1$. Theorem 2 below establishes that for $\gamma < 2$, the probability of choosing the right alternative converges to $1/2$. For $\gamma = 2$, the probability of choosing the right alternative converges to some value between $1/2$ and one. (This value is about .7412 for $r = 1$ and ϵ close to zero.) For $\gamma > 2$, the probability of choosing the right alternative converges to one.

If $C'(0) = 0$ and $C''(0) = c \in (0, \infty)$, let $k(\epsilon, r, C)$ be the solution to

$$k/\phi(k) = 4(r/c)(1 - 2\epsilon),$$

where ϕ is the standard normal density. Note that $k(\epsilon, r, C) \in (0, \infty)$, and moreover, $k(\epsilon, r, C)$ is increasing in r/c and decreasing in ϵ. As we will see below, $k(\epsilon, r, C)$ is an indicator of the information held by the electorate in large elections. It is equal to the limit of the product of the information acquired by each individual and the square root of the size of the electorate, multiplied by a constant term.

We have

Theorem 2
(i) If $C'(0) = C''(0) = 0$, the probability of choosing the right alternative converges to one as the size of the electorate increases.
(ii) If $C'(0) = 0$ and $C''(0) = c \in (0, \infty)$, the probability of choosing the right alternative converges to $\Phi(k(\epsilon, r, C))$, where Φ is the standard normal distribution.
(iii) If $C'(0) > 0$ or $C''(0) = \infty$, the probability of choosing the right alternative converges to $1/2$.

We prove the theorem via two lemmas. In the two lemmas we write x_n to represent the value of x_M (as defined in the previous section) for a given n. We know from the previous section that if $C'(0) = 0$, then x_n is positive but converges to zero as n grows to infinity. The first lemma tells us how fast x_n converges to zero in each of the three cases of the theorem. The second lemma uses a version of the central limit theorem to establish the desired results. A direct application of the central limit theorem is not possible because the distribution representing the decision of a given voter changes with n. Instead, we use a normal approximation result for finite samples, the Berry-Esseen theorem.

Lemma 1
(i) If $C'(0) = C''(0) = 0$, then $n^{1/2}x_n$ goes to $+\infty$ as n grows arbitrarily large.
(ii) If $C'(0) = 0$ and $C''(0) = c < \infty$, then
$$\lim_{n\to\infty} n^{1/2}x_n = k(\epsilon, r, C)/(2\sqrt{2}(1 - 2\epsilon)).$$
(iii) If $C'(0) = 0$ and $C''(0) = \infty$, then $\lim_{n\to\infty} n^{1/2}x_n = 0$.

Proof. For large n, if $C'(0) = 0$ then x_n is given by the solution to $G(x_n) = 0$ or

$$\frac{(2n)!}{n!n!}\left(\frac{1}{4} - (1 - 2\epsilon)^2 {x_n}^2\right)^n r = C'(x_n).$$

Letting $y_n = n^{1/2}x_n$ we get

$$\frac{(2n)!}{n!n!}\left(\frac{1}{4} - (1 - 2\epsilon)^2 \frac{{y_n}^2}{n}\right)^n r = C'(n^{-1/2}y_n).$$

Using the mean value theorem for C' and rearranging slightly we have

$$\text{(2)} \qquad \frac{(2n)!}{n!n!}\frac{n^{1/2}}{2^{2n}}\left(1-4(1-2\epsilon)^2\frac{{y_n}^2}{n}\right)^n r = y_n\, C''(\xi_n)$$

for some ξ_n between zero and $n^{-1/2}y_n$.

Note that

$$\frac{(2n)!}{n!n!}\frac{n^{1/2}}{2^{2n}} \longrightarrow \pi^{-1/2}$$

(from Stirling's formula) and

$$0 < \left(1-4(1-2\epsilon)^2\frac{{y_n}^2}{n}\right)^n < 1$$

(because $0 < y_n < n^{1/2}$).

Now consider the case $C'(0) = C''(0) = 0$. Suppose that along some subsequence y_n converges to a finite $L \geq 0$. Then, along the subsequence the right hand side of equation (2) converges to zero. However, the left hand side converges to a positive number, as can be seen from the fact that

$$\left(1-4(1-2\epsilon)^2\frac{{y_n}^2}{n}\right)^n \longrightarrow \exp\{-4(1-2\epsilon)^2L^2\}$$

(see e.g. Durrett 1991, p. 94). Thus, y_n diverges to $+\infty$, which establishes case (i).

Consider the case $C'(0) = 0$ and $C''(0) = c < \infty$. Suppose that along some subsequence y_n converges to a finite $K \geq 0$. Following the steps of the previous case, we get that K must satisfy $\pi^{-1/2}\exp\{-4(1-2\epsilon)^2K^2\}r = Kc$ or, equivalently, $K = k(\epsilon, r, C)/(2\sqrt{2}(1-2\epsilon))$. It remains to show that along no subsequence y_n diverges to $+\infty$. To see this, note that the right hand side of equation (2) grows without bound if y_n goes to infinity, while for any positive δ, the left hand side is smaller than $(\pi^{-1/2}+\delta)r$ for n large enough. This establishes case (ii).

Finally, consider the case $C'(0) = 0$ and $C''(0) = \infty$. If along some subsequence y_n converges to a finite $L > 0$ or diverges to $+\infty$, the right hand side of equation (2) grows without bound, while the left hand side is bounded by the argument above. □

Lemma 2 *Suppose that $C'(0) = 0$. If $\lim_{n\to\infty} n^{1/2}x_n = K < \infty$, the probability of choosing the right alternative converges to $\Phi(2\sqrt{2}(1-2\epsilon)K)$, where Φ is the standard normal distribution. If $n^{1/2}x_n$ diverges to $+\infty$, the probability of choosing the right alternative converges to one.*

PROOF. Suppose the state is z_A (similar calculations hold if the state is z_B). Given the equilibrium strategy described in Theorem 1(i), the event of a given voter voting for A in state z_A corresponds to a Bernoulli trial with probability of success

$$(1-2\epsilon)(1/2+x_n)+\epsilon = 1/2+(1-2\epsilon)x_n.$$

For $n = 1, 2, \ldots$ and $i = 1, \cdots, 2n+1$ define the random variables

$$V_i^n = \begin{cases} 1/2-(1-2\epsilon)x_n & \text{if voter } i \text{ votes for } A, \\ -1/2-(1-2\epsilon)x_n & \text{if voter } i \text{ votes for } B. \end{cases}$$

For each n, the random variables V_i^n are iid. Moreover,

$$\begin{aligned} &E(V_i^n) = 0, \\ &E((V_i^n)^2) = 1/4-(1-2\epsilon)^2x_n^2, \quad \text{and} \\ &E(|V_i^n|^3) = 2\left(1/16-(1-2\epsilon)^4x_n^4\right). \end{aligned}$$

Let F_n stand for the distribution of the normalized sum

$$(V_1^n+\cdots+V_{2n+1}^n)/\sqrt{E((V_i^n)^2)(2n+1)}.$$

Note that A loses the election if it obtains n or fewer votes, that is, if

$$V_1^n+\cdots+V_{2n+1}^n+(2n+1)(1/2+(1-2\epsilon)x_n) \le n$$

or equivalently

$$V_1^n+\cdots+V_{2n+1}^n \le -1/2-(2n+1)(1-2\epsilon)x_n.$$

Then, the probability of A winning the election is $1-F_n(J_n)$, where

$$J_n = \frac{-1/2-(2n+1)(1-2\epsilon)x_n}{\sqrt{E((V_i^n)^2)(2n+1)}}.$$

Now, from the Berry-Esseen theorem (see Feller 1971, p. 542 or Durrett 1991, p. 106), for all w,

$$|F_n(w) - \Phi(w)| \leq \frac{3E(|V_i^n|^3)}{E((V_i^n)^2)^{3/2}\sqrt{2n+1}}.$$

The right hand side of the equation above converges to zero as n goes to infinity, so we obtain an increasingly good approximation using the normal distribution even though the distribution of V_i^n changes with n. Thus,

$$\lim_{n\to\infty} |F_n(J_n) - \Phi(J_n)| = 0.$$

If $\lim_{n\to\infty} n^{1/2}x_n = K < \infty$, then J_n converges to $-2\sqrt{2}(1-2\epsilon)K$. Since Φ is continuous,

$$\lim_{n\to\infty} |\Phi(J_n) - \Phi(-2\sqrt{2}(1-2\epsilon)K)| = 0.$$

Thus, the probability of A winning converges to $1 - \Phi(-2\sqrt{2}(1-2\epsilon)K)$. The desired result follows from symmetry.

If $n^{1/2}x_n$ goes to infinity with n, then J_n goes to $-\infty$. Thus, for arbitrarily large L, the probability of A winning the election is larger than $1 - F_n(-L)$ for n large enough. Using the normal approximation above we can see that the probability of A winning must go to one. □

5 The Winning Margin

Define the *winning margin* to be a random variable representing the difference between the number of votes for the winner and the number of votes for the loser, divided by $2n+1$. In this section we show that, if $C'(0) = 0$, the winning margin is likely to be close to zero for large electorates. In other words, elections with information acquisition tend to be very close. Intuitively, information acquisition requires that the probability of a voter being pivotal should not decline too fast. Otherwise, voters would lose the incentive to acquire costly information.

From the previous section, we know that, if $C'(0) = C''(0) = 0$, the probability that the right alternative wins the election goes to one as the size of the electorate increases. Theorem 2 and Theorem 3 below imply that, if

$C'(0) = C''(0) = 0$, the percentage of votes for the right alternative will be very likely to be barely above 1/2. The reason is that the distribution of the percentage of votes for the right alternative concentrates very fast around its central terms, near $1/2 + (1-2\epsilon)x_n$, with x_n going to zero as n increases.

We have

Theorem 3 *If $C'(0) = 0$, then for any $\kappa > 0$ the probability that the winning margin is larger than κ converges to zero as the size of the electorate increases.*

PROOF. Suppose the state is z_A (similar calculations hold if the state is z_B). Using the notation of the proof of lemma 2, the number of votes for A is given by

$$V_1^n + \cdots + V_{2n+1}^n + (2n+1)(1/2 + (1-2\epsilon)x_n).$$

Then, the winning margin is

$$\left| \frac{2\left(\sum_{i=1}^{2n+1} V_i^n + (2n+1)(1/2 + (1-2\epsilon)x_n)\right) - (2n+1)}{2n+1} \right|$$

or equivalently,

$$2\left|\tfrac{1}{2n+1}\textstyle\sum_{i=1}^{2n+1} V_i^n + (1-2\epsilon)x_n\right|.$$

Therefore, the probability that the winning margin is smaller or equal to κ is equal to $F_n(D_n) - F_n(I_n)$, where

$$D_n = \frac{(2n+1)(\kappa/2 - (1-2\epsilon)x_n)}{\sqrt{E((V_i^n)^2)(2n+1)}}$$

and

$$I_n = \frac{(2n+1)(-\kappa/2 - (1-2\epsilon)x_n)}{\sqrt{E((V_i^n)^2)(2n+1)}}.$$

Note that D_n goes to $+\infty$ and I_n goes to $-\infty$ with n. Following the last steps of the proof of lemma 2, we have that the probability that the winning margin is smaller or equal to κ must go to one. $\square$

Theorem 3 is reminiscent of a similar result by Feddersen and Pesendorfer (1997). In their work, the tightness of the electoral race is brought about by the fact that a vanishing fraction of voters takes into account their private information when casting a vote. That is, only a small fraction of voters takes informative actions in large elections. In our model, the fraction of swing voters is constant because of our assumption that moderate voters have common preferences. However, in large elections, the actions of individual voters carry very little information.

6 Free Information

In this section, we modify the model presented in Section 2 by allowing voters to receive some free information. In particular, we now assume that the probability of receiving the signal s_A in state A is equal to the probability of receiving signal s_B in state B and is given by $1/2 + \delta + x$ for some $\delta \in (0, 1/2)$. Acquiring additional information quality $x \in [0, 1/2 - \delta]$ has a utility cost given by a strictly increasing, strictly convex and twice differentiable function $C(x)$ with $C(0) = 0$. Surprisingly perhaps, we get that voters acquire information under the same conditions as in the model without free information.

Define

$$\tilde{G}(x) = \frac{(2n)!}{n!n!}\left(\frac{1}{4} - (1-2\epsilon)^2(x+\delta)^2\right)^n r - C'(x),$$

and let

$$\tilde{x}_M = \begin{cases} 0 & \text{if } \tilde{G}(0) \leq 0 \\ 1/2 - \delta & \text{if } \tilde{G}(1/2-\delta) \geq 0 \\ \tilde{G}^{-1}(0) & \text{otherwise.} \end{cases}$$

Note that $\tilde{x}_M > 0$ for arbitrarily large n if and only if $C'(0) = 0$.

We have

Theorem 4 *In the model with free information, there is a unique voting equilibrium. In this equilibrium, the pure strategy given by* $a_x(t_M) = \tilde{x}_M$, $a_v(t_M, s_A) = A$ *and* $a_v(t_M, s_B) = B$ *is played with probability one.*

The proof just follows that of Theorem 1. The only difference is that moderate voters vote according to their signals even if they acquire no additional information. Of course, whether or not there is information acquisition becomes asymptotically irrelevant in the presence of free information. Since the probability of a voter casting a vote for the right alternative is bounded below by $(1-2\epsilon)(1/2+\delta)+\epsilon = 1/2+\delta(1-2\epsilon)$, the probability of choosing the right alternative must converge to one.

7 Conclusion

The representation of public opinion in our model is very sparse. There are no media, interest groups or other political organizations; only voters who may choose to acquire and process some arbitrarily poor information. In this setting, we have shown that the electorate as a whole may be much better informed than individual voters. In light of this, we believe that the implications of "rational ignorance" for the behavior of the electorate as a whole may have been exaggerated.

Akerlof and Yellen (1985) have argued that a small amount of nonmaximizing behavior by agents is capable of causing large changes in economic equilibria, including in some contexts large welfare losses. In the political environment we study, a small deviation from rationality by voters – ignoring completely the effects of a single opinion – would have important negative effects on the responsiveness of collective decision making to the interests of the majority. However, deviations from strictly rational beliefs may be as likely to occur in the direction of overestimating the importance of a single opinion as in the direction of underestimating it. This is a matter better left for future research.

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